Math 447 - October 7, 2011 - Quiz 17 Solutions

Name:_____

On a television game show, the host showed a contestant *four* curtains. Behind one of the curtains was a nice prize (perhaps a new car) and behind the other three curtains were worthless prizes (duds). The contestant chose a curtain, expecting to receive the prize behind it, and hoping it was the nice one. Before showing the contestant what was behind the initially chosen curtain, the host would open one of the other curtains and show the contestant one of the duds. He could always do this because he knew the curtain hiding the good prize, and every time the game was played, he showed a dud prize.

He then offered the contestant the option of changing from the initially selected curtain to one of the two remaining unopened ones.

1. What is the probability of winning the good prize by choosing to switch curtains?

We label the curtains by the prizes behind them, so the sample space of outcomes of the initial choice is $\{G, D_1, D_2, D_3\}$. Each choice has probability $1/4$, as the contestant has no information about what is behind the curtains.

The results from switching curtains can be divided into two cases. In the first case, the initial selection was G, which has probability $1/4$, and switching curtains loses. In the second case, the initial selection was D_i for some i, and switching eliminates D_i and some other D_j, the curtain revealed by the host. Thus, in the second case, which has probability $3/4$, the guest wins half the time.

Thus the probability of winning by switching is $3/8$.

2. Suppose now instead of four curtains there are n curtains, but the game is played in the same way. Find the probability of winning the prize by choosing to switch curtains as a function of n.

Repeat the same analysis: the relevant changes are that the second case has probability $(n-1)/n$ and the probability of winning in the second case is $1/(n-2)$. Thus the answer is

$$\frac{n-1}{n(n-2)}.$$

Note that this evaluates to the previous answer when $n = 4$.