1. The figure to the left shows the pole-zero diagram of a continuous-time (analog) filter.

   (a) What kind of filter is this?

   (b) How was this filter designed? Be as specific as you can about the design parameters.

   (c) Outline the steps and parameters required to convert this into a discrete-time highpass filter with a cutoff frequency $\omega_c = 0.5$ rad/samp. Say as much as you can about the characteristics of this resulting filter.

2. $h_1[n]$ is an $(N+1)$-point FIR filter defined for $n = 0..N$ as:

   $$h_1[n] = \sin\left(\pi \frac{(n - \frac{N}{2})}{4}\right) / \pi \left(n - \frac{N}{2}\right)$$

   $h_2[n]$ is also an $(N+1)$-point FIR filter designed with the Remez exchange algorithm to be a low-pass filter with a passband edge at $0.2\pi$ and a stopband edge at $0.3\pi$, and equal error weighting in pass and stop bands.

   (a) Sketch, as best you can, the impulse response, magnitude response, phase response, and pole-zero plot for filter $h_1[n]$ for $N = 14$. Where there are ambiguities in your sketch, discuss what they are.

   (b) Repeat part (a) for filter $h_2[n]$. Emphasize any similarities or differences between the corresponding plots in part (a).

   (c) $h_3[n]$ is the same as $h_1[n]$ except that $h_3[N/2] = -3/4$. Repeat part (a) for this new filter.
3. In the $z$-plane, one pair (pair A) of conjugate roots lie at $z = r_0 e^{\pm j\omega_0}$ and a second pair (pair B) lie at $z = \frac{1}{r_0} e^{\pm j\omega_0}$. (Assume $0 < r_0 < 1$.) Each of A and B can be a pair of zeros or a pair of poles.

   (a) Sketch the magnitude and phase responses for each of the possible filters formed by these choices.

   (b) If we have a signal that has been processed by one of these filters, how might we be able to transform it into the result of feeding the original signal into another of these filters? Describe and discuss for each possible pair of filters.

4. $x[n]$ is an $M$-point sequence, where $M = 2^L$.
   $x_0[n]$ and $x_1[n]$ are two $N$-point sequences, where $N = M/2$.
   $x_0[n] = x[n]$ and $x_1[n] = x[n + N]$, both for $n = 0 \ldots N - 1$.

   (a) If you are given only $X_0[k]$ and $X_1[k]$, the DFTs of the two $N$-point sequences, shown that $X[k]$, the DFT of the $M$-point sequence, obeys

   $$X[k] = X_0[k/2] + X_1[k/2]$$

   for even $k$. Derive an expression for $X[k]$ in terms of $X_0[k]$ and $X_1[k]$ for odd $k$.

   (b) If the number of (complex) multiplies to perform a $N$-point DFT is approximately $\frac{N}{2} \cdot \log_2 N$, compare the computational cost of finding $X[k]$ using the relationships from part (a) with finding $X[k]$ by first reconstructing $x[n]$ from $X_0[k]$ and $X_1[k]$ then taking its DFT.