Chem340 HW1 Key (1/22/2011)

P1.2) Consider a gas mixture in a 2.00-dm³ flask at 27.0°C. For each of the following mixtures, calculate the partial pressure of each gas, the total pressure, and the composition of the mixture in mole percent:

- **a.** 1.00 g H_2 and 1.00 g O_2
- **b.** 1.00 g N_2 and 1.00 g O_2
- **c.** 1.00 g CH₄ and 1.00 g NH₃

To calculate the partial pressures we use the ideal gas law:

a)

$$p_{H_2} = \frac{n_{H_2} R T}{V} = \frac{m_{H_2} R T}{M_{H_2} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^5 \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^5 \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^5 \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^5 \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^{-5} \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^{-5} \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^{-5} \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^{-5} \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{6.18 \times 10^{-5} \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (2.02 \times 10^{-3} \text{ kg mol}^{-1})}$$

$$p_{o_2} = \frac{n_{o_2} R T}{V} = \frac{m_{o_2} R T}{M_{o_2} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (32.0 \times 10^{-3} \text{ kg mol}^{-1})} = 3.90 \times 10^5 \text{ Pa}$$

 $p_{total} = p_{H_2} + p_{O_2} = 6.57 \times 10^5 \text{ Pa}$

b)

$$p_{N_2} = \frac{n_{N_2} R T}{V} = \frac{m_{N_2} R T}{M_{N_2} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (28.02 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{4.45 \times 10^4 \text{ Pa}}{4.45 \times 10^4 \text{ Pa}}$$
$$p_{O_2} = \frac{n_{O_2} R T}{V} = \frac{m_{O_2} R T}{M_{O_2} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (32.0 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{3.90 \times 10^5 \text{ Pa}}{4.45 \times 10^4 \text{ Pa}}$$

 $p_{total} = p_{H_2} + p_{O_2} = 8.35 \times 10^4 \text{ Pa}$

$$p_{NH_3} = \frac{n_{NH_3} R T}{V} = \frac{m_{NH_3} R T}{M_{NH_3} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (17.03 \times 10^{-3} \text{ kg mol}^{-1})} = \frac{7.32 \times 10^4 \text{ Pa}}{(2.00 \times 10^{-3} \text{ m}^3) \times (17.03 \times 10^{-3} \text{ kg mol}^{-1})}$$

$$p_{CH_4} = \frac{n_{CH_4} R T}{V} = \frac{m_{CH_4} R T}{M_{CH_4} V} = \frac{(0.001 \text{ kg}) \times (8.314472 \text{ J } \text{K}^{-1} \text{ mol}^{-1}) \times (300.15 \text{ kg})}{(2.00 \times 10^{-3} \text{ m}^3) \times (16.04 \times 10^{-3} \text{ kg mol}^{-1})} = 7.77 \times 10^4 \text{ Pa}$$

 $p_{total} = p_{H_2} + p_{O_2} = 1.51 \times 10^5 \text{ Pa}$

$$\begin{split} & \operatorname{mol} \% \operatorname{NH}_{3} = 100 \times \frac{\operatorname{mol} \operatorname{NH}_{3}}{\left(\operatorname{mol} \operatorname{NH}_{3} + \operatorname{mol} \operatorname{CH}_{4}\right)} \\ & = 100 \times \frac{\left(\frac{0.001 \, \text{kg}}{17.03 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right)}{\left(\left(\frac{0.001 \, \text{kg}}{16.04 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right) + \left(\frac{0.001 \, \text{kg}}{17.03 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right)\right)} = \underline{48.5 \%} \\ & \operatorname{mol} \% \operatorname{CH}_{4} = 100 \times \frac{\operatorname{mol} \operatorname{CH}_{4}}{\left(\operatorname{mol} \operatorname{NH}_{3} + \operatorname{mol} \operatorname{CH}_{4}\right)} \\ & = 100 \times \frac{\left(\frac{0.001 \, \text{kg}}{17.03 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right)}{\left(\left(\frac{0.001 \, \text{kg}}{16.04 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right) + \left(\frac{0.001 \, \text{kg}}{17.03 \times 10^{-3} \, \text{kg} \, \text{mol}^{-1}}\right)\right)} = \underline{51.5 \%} \end{split}$$

P1.3) Approximately how many oxygen molecules arrive each second at the mitochondrion of an active person? The following data are available: oxygen consumption is about 40. mL of O_2 per minute per kilogram of body weight, measured at T = 300. K and P = 1.0 atm. An adult with a body weight of 64 kg has about 1×10^{12} cells. Each cell contains about 800. mitochondria. With pV = nRT the number of moles per minute and per kg of body weight is:

$$n = \frac{p V}{R T} = \frac{(101325 Pa) \times (4.0 \times 10^{-5} m^3)}{(300 K) \times (8.314472 J K^{-1} mol^{-1}) \times (1 min)} = 1.6249 \times 10^{-3} mol min^{-1}$$

For a body weight of 64 kg and per second the number of moles is:

n =
$$(1.6249 \times 10^{-3} \text{ mol min}^{-1}) \times (64 \text{ kg}) \times (\frac{1 \text{ min}}{60 \text{ s}}) = 1.7332 \times 10^{-3} \text{ mol}$$

Converting to molecules:

molecules $O_2 = (1.7332 \times 10^{-3} \text{ mol}) \times (6.02214 \times 10^{23} \text{ molecules mol}^{-1}) = 1.04 \times 10^{21} \text{ molecules Wit}$ h 1 × 10¹² cells in a 64 kg body, and 800 mitochondria in each cell:

molecules
$$O_2 = \frac{(1.04 \times 10^{21})}{(1.0 \times 10^{12}) \times (800)} = \underline{1.25 \times 10^6}$$
 molecules

P1.4) In a normal breath, about 0.5 L of air at 1.0 atm and 293 K is inhaled. About 25.0% of the oxygen in air is absorbed by the lungs and passes into the bloodstream. For a respiration rate of

18 breaths per minute, how many moles of oxygen per minute are absorbed by the body? Assume the mole fraction of oxygen in air is 0.21. Compare this result with Example Problem 1.1. We use the ideal gas law to calculate the number of moles of air inhaled every minute:

$$n_{air} = \frac{p V}{R T} = \frac{(101325 Pa) \times (5.0 \times 10^{-4} m^3)}{(2930 K) \times (8.314472 J K^{-1} mol^{-1})} = 0.0208 mol$$

We then use the mole fraction to determine the number of moles of O₂ for 18 breaths:

$$n_{O_2} = (0.0208 \text{ mol}) \times (0.21) \times (0.25) \times (18 \text{ min}^{-1}) = 0.01966 \text{ mol min}^{-1}$$

P1.7) A rigid vessel of volume 0.500 m³ containing H₂ at 20.5°C and a pressure of 611×10^3 Pa is connected to a second rigid vessel of volume 0.750 m³ containing Ar at 31.2°C at a pressure of 433×10^3 Pa. A valve separating the two vessels is opened and both are cooled to a temperature of 14.5°C. What is the final pressure in the vessels?

We need to first calculate the number of moles of H₂ and Ar using the ideal gas law:

$$n_{H_2} = \frac{p V}{R T} = \frac{(611 \times 10^3 Pa) \times (0.500 m^3)}{(273.15 K + 20.5 K) \times (8.314472 J K^{-1} mol^{-1})} = 125 mol$$

$$n_{Ar} = \frac{p V}{R T} = \frac{(433 \times 10^{3} Pa) \times (0.750 m^{3})}{(273.15 K + 31.2 K) \times (8.314472 J K^{-1} mol^{-1})} = 128 mol$$

Using the final number of moles, the final pressure at 14.5°C is then:

$$p = \frac{n_{\text{total}} \text{ R T}}{V_{\text{total}}} = \frac{(128 \text{ mol} + 125 \text{ mol}) \times (273.15 \text{ K} + 14.5 \text{ K}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1})}{(0.500 \text{ m}^3 + 0.750 \text{ m}^3)} = 4.84 \times 10^5 \text{ Pa}$$

P1.9) At T = 293 K and at 50.% relative humidity, the pressure of water vapor in equilibrium with liquid water is 0.0115 atm. Using the information in Problem P1.8, determine what mass of

water is inhaled per hour and the net loss of water through respiration per hour.

Using the ideal gas law, and solving for m, with the volume from P1.8 yields:

$$p V = n R T = \frac{m}{M} R T$$

$$m = \frac{M p V}{R T} = \frac{(18.02 \times 10^{-3} \text{ kg mol}^{-1}) \times (0.0115 \text{ atm}) \times (101325 \text{ Pa atm}^{-1}) \times (0.5 \text{ m}^{-3})}{(293 \text{ K}) \times (8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (1 \text{ h})} \times 0.5 = 2.15 \times 10^{-3} \text{ kg h}^{-1}$$

With the results from P1.8, the net loss of water is then:

net loss = $0.0227 \text{ kg h}^{-1} - 2.15 \times 10^{-3} \text{ kg h}^{-1} = 0.0205 \text{ kg h}^{-1}$

P1.15) Consider a 20.0-L sample of moist air at 60.°C and 1 atm in which the partial pressure of water vapor is 0.120 atm. Assume that dry air has the composition 78.0 mol % N_2 , 21.0 mol % O_2 , and 1.00 mol % Ar.

- a. What are the mole percentages of each of the gases in the sample?
- **b.** The percent relative humidity is defined as $\% \text{RH} = P_{H_2O} / P_{H_2O}^*$ where P_{H_2O} is the partial pressure of water in the sample and $P_{H_2O}^* = 0.197$ atm is the equilibrium vapor pressure of water at 60.°C. The gas is compressed at 60.°C until the relative humidity is 100%. What volume does the mixture contain now?
- **c.** What fraction of the water will be condensed if the total pressure of the mixture is isothermally increased to 200. atm?

a) mol % N₂ = 100 ×
$$\frac{p_{N_2}}{p_{\text{total}}}$$
 = 100 × $\frac{(0.78 \times 0.88 \text{ atm})}{(1.00 \text{ atm})}$ = $\underline{68.6 \%}$

mol % O₂ = 100×
$$\frac{p_{O_2}}{p_{\text{total}}}$$
 = 100× $\frac{(0.21 \times 0.88 \text{ atm})}{(1.00 \text{ atm})}$ = 18.5 %

mol % Ar = 100 ×
$$\frac{p_{Ar}}{p_{total}}$$
 = 100 × $\frac{(0.01 \times 0.88 \text{ atm})}{(1.00 \text{ atm})}$ = 0.9%

mol % H₂O = 100 ×
$$\frac{p_{H_2O}}{p_{total}}$$
 = 100 × $\frac{(0.12 \text{ atm})}{(1.00 \text{ atm})}$ = $\underline{12.0 \%}$

 $p_{_{H_2O}} V = \frac{n_{_{H_2O}} R T}{V}$

 $p_{\rm H_{2}O}^{\,\prime}$ V $^{\prime}$ = $p_{\rm H_{2}O}$ V , where the prime refers to 100% RH

$$V' = \frac{p_{H_{2}O} V}{p'_{H_{2}O}} = \frac{(0.12 \text{ atm}) \times (20.0 \text{ L})}{(0.197 \text{ atm})} = 12.2 \text{ L}$$

If all the water remained in the gas phase, the partial pressure of water at a total pressure of 200 atm would be:

 $p_{\rm H_2O} = p_{\rm total} \times mol \ fraction \ H_2O = 200 \ atm \times 0.12 = 24.0 \ atm$

However, the partial pressure of water cannot be greater than 0.197 atm, and the excess will condense. The fraction that condenses is given by:

freation condensed =
$$\frac{(24.0 \text{ atm}) - (0.197 \text{ atm})}{(24.0 \text{ atm})} = 0.992$$

P1.18) A normal adult inhales 0.500 L of air at T = 293 K and 1.00 atm. To explore the surface of the moon, an astronaut requires a 25.0-L breathing tank containing air at a pressure of 200.

atm. How many breaths can the astronaut take from this tank?

We first need to calculate the number of moles inhaled at 1 atm using the ideal gas law:

$$n_{\text{breath}} = \frac{p V}{R T} = \frac{(101325 \text{ Pa}) \times (0.0005 \text{ m}^3)}{(8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (293 \text{ K})} = 0.0208 \text{ mol}$$

Next we determine how many moles of air are in the tank:

$$n_{tank} = \frac{p V}{R T} = \frac{(200 \text{ x } 101325 \text{ Pa}) \times (0.025 \text{ m}^3)}{(8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (293 \text{ K})} = 207.96 \text{ mol}$$

Therefore, the number of breaths the astronaut can take is:

number of breaths = $\frac{n_{tank}}{n_{breath}} = \frac{(207.96 \text{ mol})}{(0.0208 \text{ mol})} = 9998$

P1.21) A sample of propane (C_3H_8) is placed in a closed vessel together with an amount of O_2 that is 3.00 times the amount needed to completely oxidize the propane to CO_2 and H_2O at constant temperature. Calculate the mole fraction of each component in the resulting mixture after oxidation assuming that the H_2O is present as a gas.

The reaction we have to consider is:

$$C_{3}H_{8}(g)+5O_{2}(g)\longrightarrow 3CO_{2}(g)+4H_{2}O(g)$$

If m moles of propane are present initially, there must be 15 m moles of O_2 . After the reaction is complete, there are 3 m moles of CO_2 , 4 m moles of H_2O , and 10 m moles of O_2 . Therefore:

$$x_{CO_2} = \frac{3 m}{17 m} = 0.176;$$
 $x_{H_2O} = \frac{4 m}{17 m} = 0.235,$ $x_{O_2} = \frac{10 m}{17 m} = 0.588$

P1.27) Calculate the pressure exerted by Ar for a molar volume of $1.42 \text{ L} \text{ mol}^{-1}$ at 300. K using the van der Waals equation of state. The van der Waals parameters *a* and *b* for Ar are 1.355 bar

 $dm^6 mol^{-2}$ and 0.0320 $dm^3 mol^{-1}$, respectively. Is the attractive or repulsive portion of the potential dominant under these conditions?

To determine what portion of the potential is dominant we need to compare the pressure predicted by the van der Waals equation of state with that predicted by the ideal gas law. The van der Waals equation of state yields:

$$p_{vdW} = \frac{R T}{(V_m - b)} - \frac{a}{V_m^2} = \frac{\left(8.314472 \times 10^{-2} \text{ bar } \text{dm}^3 \text{ K}^{-1} \text{ mol}^{-1}\right) \times (300 \text{ K})}{\left(\left(1.42 \text{ dm}^3 \text{ mol}^{-1}\right) - \left(0.0321 \text{ dm}^3 \text{ mol}^{-1}\right)\right)} - \frac{\left(1.355 \text{ bar } \text{ dm}^6 \text{ mol}^{-2}\right)}{\left(1.42 \text{ dm}^3 \text{ mol}^{-1}\right)^2} = 17.3 \text{ bar}$$

The ideal gas law gives:

$$p_{ideal} = \frac{R T}{V_{m}} = \frac{\left(8.314472 \times 10^{-2} \text{ bar } \text{dm}^{3} \text{ K}^{-1} \text{ mol}^{-1}\right) \times (300 \text{ K})}{\left(1.42 \text{ dm}^{3} \text{ mol}^{-1}\right)} = 17.6 \text{ bar}$$

Because $p_{vdW} < p_{ideal}$, the attractive part of the potential dominates.

P1.30) The barometric pressure falls off with height above sea level in the Earth's atmosphere as $P_i = P_i^0 e^{-M_i g/RT}$ where P_i is the partial pressure at the height *z*, P_i^0 is the partial pressure of component *i* at sea level, *g* is the acceleration of gravity, *R* is the gas constant, *T* is the absolute temperature, and M_i is the molecular mass of the gas. Consider an atmosphere that has the composition $x_{N_2} = 0.600$ and $x_{CO_2} = 0.400$ and that T = 300. K. Near sea level, the total pressure is 1.00 bar. Calculate the mole fractions of the two components at a height of 50.0 km. Why is the composition different from its value at sea level?

$$p_{N_{2}} = p_{N_{2}}^{\circ} e^{-\frac{M_{N_{2}} gz}{RT}}$$
$$= (0.600) \times (1.01325 \times 10^{5} \text{ Pa}) \times \text{Exp} \left[-\frac{(28.04 \times 10^{-3} \text{ kg mol}^{-1}) \times (9.81 \text{ m s}^{-1}) \times (50 \times 10^{3} \text{ m})}{(8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300 \text{ K})} \right] = 248 \text{ Pa}$$

$$p_{CO_{2}} = p_{CO_{2}}^{\circ} e^{-\frac{M_{CO_{2}} g z}{R T}}$$
$$= (0.400) \times (1.01325 \times 10^{5} \text{ Pa}) \times \text{Exp} \left[-\frac{(44.04 \times 10^{-3} \text{ kg mol}^{-1}) \times (9.81 \text{ m s}^{-1}) \times (50 \times 10^{3} \text{ m})}{(8.314472 \text{ J K}^{-1} \text{ mol}^{-1}) \times (300 \text{ K})} \right] = 7.02 \text{ Pa}$$

$$x_{CO_2} = \frac{p_{CO_2}}{(p_{CO_2} + p_{N_2})} = \frac{(7.02 \text{ Pa})}{(7.02 \text{ Pa} + 248 \text{ Pa})} = 0.028$$

 $x_{N_2} = 1 - x_{CO_2} = 0.972$

- For O₂, calculate (a) the molecular mass (b) the molecular weight (c) the mass of 1 molecule of O₂ (d) the molar mass.
 - (a) $16.00 \times 2 = 32.00$ u
 - (b) 32.00
 - (c) $32.00 \text{ u} = 32.00 \text{ g} / 6.022 \text{ x} 10^{23} = 5.314 \text{ x} 10^{23} \text{ g}$
 - (d) 32.00 g/mol
- 2. Give the (a) weight % and (b) molar % of O_2 in the air at $25^{\circ}C$

By P1.15, mol% $N_2 = 78.0\%$, mol% $O_2 = 21.0\%$, mol% Ar = 1.0%

(a)

weight%O₂ =
$$\frac{M_{O_2} \times mol\%O_2}{M_{N_2} \times mol\%N_2 + M_{O_2} \times mol\%O_2 + M_{Ar} \times mol\%Ar}$$

=
$$\frac{32.0g/mol \times 21.0\%}{28.0g / mol \times 78.0\% + 32.0g / mol \times 21.0\% + 40.0g / mol \times 1.0\%}$$

= 23.2%

(b) Molar % of O2 is the same as mol % of O2, since the volumes of O2, N2, Ar are the same.

Molar % O2 = 21%

3. What is the relationship of temperatures denoted in K (T) and C (T')? Use one equation.

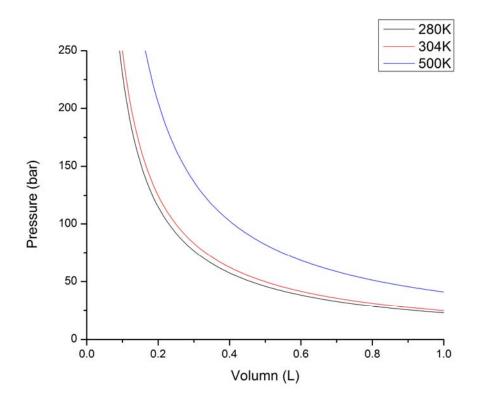
 $T(K) = 273.15 + T'(^{\circ}C)$

4. In P1.15, what is the partial pressure of Ar for 1.00 atm of air at 60° C?

$$p_{Ar} = p_{Air} \cdot 1.0\% = (p_{tot} - p_{water}) \cdot 0.010$$
$$= (1.00atm - 0.120atm) \cdot 0.010$$
$$= 8.8 \times 10^{-3} atm$$

5. Plot P (in bar) for a varied V (0.05 < V < 1.0 L) for one mole of air at 60°C?

$$pV = nRT$$
$$p = \frac{nRT}{V} = 1 \text{mol} \cdot 0.08314 \text{bar} \cdot \text{L} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \cdot \frac{T}{V}$$
$$= 0.08314 \frac{T}{V}$$



6. Plot P for a varied V (0.05 < V < 1.0 L) for one mole of non-ideal gas, CO₂ at T = 280 K, 304 K, and 500 K using equation 1.12 in page 9 and values of a and b in Table 1.3 (Appendix B). Display 3 graphs in one figure. Present the equations you used to draw the graph and explain how you generated each equation. What is the major difference from the graphs in 5. Use bar for the pressure unit.

$$p = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

= $nR\frac{T}{V - nb} - n^2 a \cdot \frac{1}{V^2}$
= $1 \text{mol} \cdot 0.08314 \text{bar} \cdot \text{L} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \cdot \frac{T}{V - 1 \text{mol} \cdot 0.0429 \text{L} \cdot \text{mol}^{-1}} - (1 \text{mol})^2 \cdot 3.658 \text{bar} \cdot \text{L} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \cdot \frac{1}{V^2}$
= $0.08314 \frac{T}{V - 0.0429} - 3.658 \frac{1}{V^2}$

The major difference is at the low volume region, or high pressure region. In this range, the CO2 are highly compressed. The molecular interaction is significant higher than nor-

mal case, and it influence volume significantly. So we see the valley around 0.1 L at 280 K. When the temperature goes up to 500 K, the thermal motion is more vigorous, and it breaks the interaction between CO_2 molecules, in this situation, CO_2 behaves more like ideal gas. That is why we do not see the valley, it is more like ideal gas curve, in P5.

